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ABSTRACT

External representations have a great impact on what and how students learn. One key manner in which environments can operate upon novices' knowledge is through helping them ground their problem solving in an understanding of the situation embodied by the problem. In this paper, students' difficulties in microeconomics problem solving were investigated, along with the ways traditional representations such as supply and demand graphs interfere with students' situational understanding. Fourteen gifted students in grades 7 through 9 were videotaped solving a few economics problems similar to those found in typical introductory economics courses. How they used the diagrams to help themselves construct an understanding of the situation was studied. It was found that students who experienced minor impasses relied on the notation to overcome them, even though that sometimes led the students into worse errors. In contrast, when the notation did not offer any support for the reasoning, students fell back on reasoning about the situation in the world and were often more successful. Based on these results, it is argued that not only must the initial problem solving in a domain be based on novices' understanding of the world, but also that the notation used for the problem solving must encourage students to use this understanding to guide planning and to overcome impasses throughout the problem solving. Eight figures illustrate study data. (Contains 27 references.) (Author/SLD)

Using informal knowledge in formal domains: Intuitions and notations sometimes clash

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RUNNING HEAD: INTUITIONS AND NOTATIONS

Abstract

External representations have great impact upon what students learn and how they go about it. Such a notation can even interfere with student learning. One key manner in which environments can operate upon novices' knowledge is through helping them ground their problem solving in an understanding of the situation embodied by the problem. In this paper, we investigate students' difficulties in microeconomics problem solving and the ways traditional representations such as supply and demand graphs interfere with students' situational understanding. We videotaped a group of young students solving a few economics problems similar to those found in typical introductory economics courses, and analyzed how they used the diagrams to help them construct an understanding of the situation. We found that students who experienced minor impasses relied upon the notation to overcome it, even though it sometimes led the students into worse errors. In contrast, when the notation did not offer any support for the reasoning, students fell back upon reasoning about the situation in the world and were often more successful. Based upon these results, we argue that not only must initial problem solving in a domain be based upon novices' understanding of the world, but also that the notation used for the problem solving must encourage students to use this understanding to guide planning and to overcome impasses throughout the problem solving.

A great deal of research has focused upon problem solving methods used by novices and experts (e.g., Anderson, 1983, 1990; diSessa, 1988; Newell, 1990; VanLehn, 1988). In general, students' problem solving is influenced by two different types of reasoning. The first is to rely upon preexisting understanding of the domain, notions that are learned in everyday life that are applicable to formal problem solving, such as the idea of division as cutting up an object into equal parts for sharing among a group (Confrey, 1990). The second source seems far less important: students are also strongly influenced by strategies suggested by the form of the notation used when solving the problems. However, we will argue that the effects of notations are no less important than those of the preexisting intuitions on problem solving. After describing these two influences, we will present a study that highlighted situations where students relied on notational strategies or their intuitions and describe the outcomes of each.

The role prior conceptions play in learning formal domains has been investigated under the label *constructivism* (e.g., Confrey, 1990; diSessa, 1988). Despite educators' view of children as devoid of knowledge, students actually enter instructional situations with well developed belief systems about causality and operations in many mathematical domains, from arithmetic to calculus to programming (Confrey, 1990). These belief structures are very situation specific They describe the behavior and relationships of individual objects, rather than being global theories of a domain. Thus, students may hold different conceptions that are actually in conflict with one another without realizing that they do so, since each conception only arises in its particular situation.

These student conceptions are often incorrect and incomplete. However, instruction that leads students to recognize and repair incorrect or incomplete belief structures, thereby

allowing students to learn the new domain based upon what they already know, can be very successful (diSessa, 1988). Thus, one important goal of instruction should be to build upon this intuitive understanding of a domain.

Instruction, unfortunately, often does not capitalize upon students' preexisting knowledge (diSessa, 1988). For example, people have well-articulated, although incorrect, knowledge of economics (Salter, 1986). Helping students construct new knowledge to correct and extend their preconceptions is a profitable pedagogical strategy. However, this preexisting knowledge may actually interfere with traditional problem solving in economics, because students are not sure of what knowledge is relevant and what is not (Strober & Cook, 1992).

It seems paradoxical that we are arguing, on the one hand, that student conceptions are crucial to formal learning, but, on the other hand, showing data that suggests student conceptions interfere with learning. The resolution of this paradox requires thinking about how novices solve problems in a domain such as economics.

Many traditional microeconomics problems come in two parts (e.g., Samuelson, 1980). Solving the first part requires creating and then manipulating a supply and demand graph. This sort of graph, depicted in Figure 1, represents the total market desire to purchase a certain good at some range of prices as well as the amount of that good that its producers would create at the same range of price levels. The graph in Figure 1 is an example of what would be created by microeconomics students. Not only is a graph a solution in and of itself, but students use the graph to note reasoning related to the market changes, such as increases in the number of producers of a good or market equilibrating forces in action. Later in this section we will describe some of the manipulations that often occur while generating a solution using a supply and demand graph.

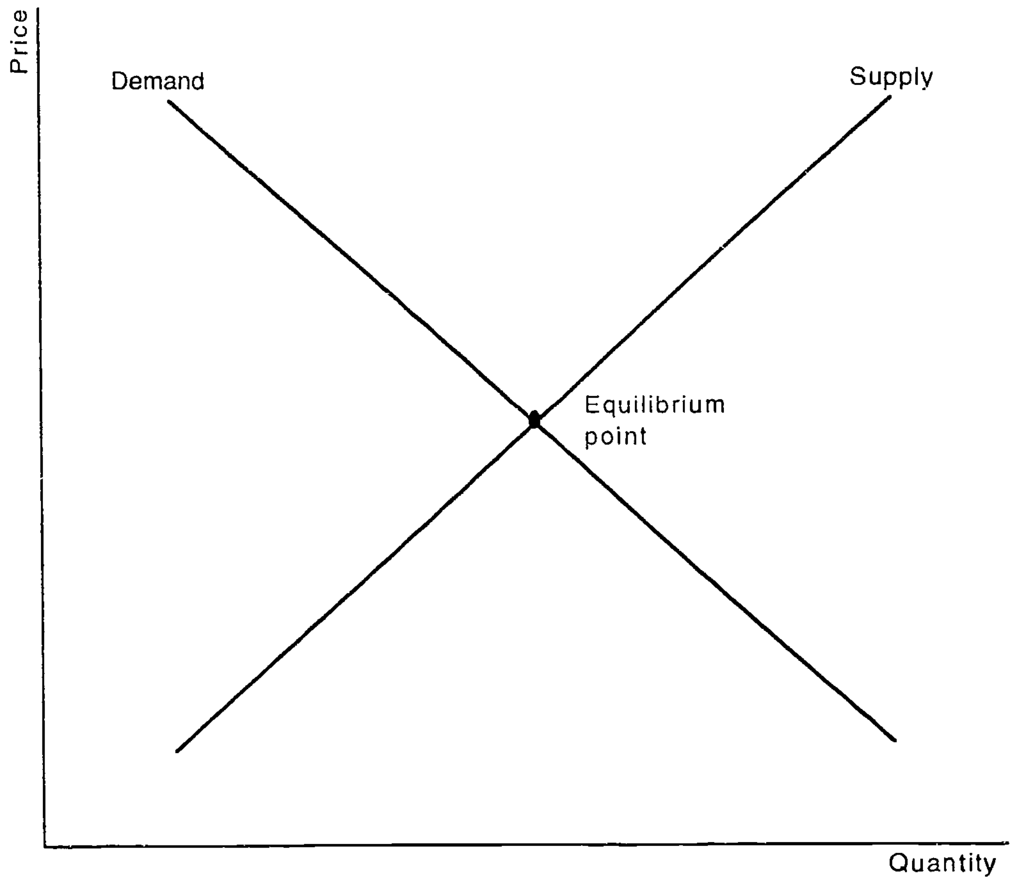

Figure 1: A short-term supply and demand graph showing the equilibrium Q_D and Q_S.

The second part of a traditional microeconomics problem would consist of algebraic manipulations of equations that model the supply curve and the demand curve in Figure 1. For example, students might be asked to use equations for supply and demand to determine the price at which there are as many goods produced as demanded; this situation is called equilibrium.

Both the supply and demand graph and the algebraic equations are notations that must be used to solve the problems. A solution to the first part of a problem is a graph, and, to the second part, a number. Each portion has certain constraints about what can and can not be done that the student must remember. For example, any number added to one side of an equation must be added to the other side, and a price change affects both supply and demand rather than one or the other. These constraints may pose difficulties for students.

In fact, notations may not only pose difficulties for students, but may even lead them into poor reasoning strategies. For example, two column proofs are a major component of geometric problem solving. Anderson, Boyle, and Yost (1986) argued that the linear, step by step, two column notation used for proofs leads students to believe that reasoning about the problems should also be linear from given information to the goals with no extraneous information. In fact, search through geometry problem spaces from the given information to goals and backwards from the goals to the the given information is important, even if it creates dead end paths (Anderson et al., 1986).

The two column proof was not intended as a reasoning method, but rather as a means of communicating an answer. However, novices not only reason about the notation to create a solution, the notation affects their reasoning. Thus, not only are they reasoning about a problem, using a notation, novices are reasoning about the notation! The notation is

suggesting a style of problem solving, and potentially could even suggest problem solving operators to apply or legislate against the application of others. Students do not tend to work backwards from the goals in a two column proof, even though they know operators that could be applied to the goal state. Notations used in problem solving may therefore constrain student problem solving options in various situations. Perhaps these constraints protect the student from extraneous complexity, but they may also lead the student to believe that certain operations should not or can not be performed.

In fairness, the designers of the two column proof did not intend to constrain students' problem solving options. They presumably intended students to search through the problem spaces and then translate their final answer into a two column format, and many students may in fact do this translation. But this translation leads to difficulties in and of itself. The additional cognitive load imposed by this translation results in students having more difficulty with the domain than students who did not have to do this translation (cf., Sweller, 1988; Trafton & Reiser, 1993). Notations are powerful and expressive when used by experts. However, they can lead novices to use using poor reasoning strategies and can constrain problem solving without making the constraints explicit.

In fact, notations are not sufficient in and of themselves to support experts when they attempt to solve very difficult problems. Experts solving such difficult problems use both formal notations and intuitive causal descriptions of behavior in the domain. In fact, a central component of their expertise is knowing when to move back and forth between the informal and notational problem solving (Bauer & Reiser, 1990).

Experts use their understanding of the situations to help them choose which of many possible solution methods to employ and help them identify errors in their notational solu-

tions. This is an important point – experts not only reason about the notation per se, but also the referents of the various notational constructs, and reasoning about the referents constrains the notational solutions.

If experts use both notational and causal knowledge when solving problems, perhaps novices should as well. Indeed, White (1993) and Nathan, Kintsch, and Young (1992) argue that significant pedagogical advantages arise from leading students to reason not only about algebraic manipulations, but also about the situations embodied by the equations. As with experts, the situations to which the problems refer can constrain the the notational solutions. A problem facing novices may have more than one potential answer, or a notational error may lead a student to an answer different from the intended one. In either case, reasoning about the situation in the problem could help students determine which of the possible answers to select or could indicate that an error has occurred. For example, if the situation described in a geometry problem asks for the length of a shadow cast by a fully grown tree at noon, the shadow should probably not be longer than the tree's height.

While it may be true that novices should reason about the situation when solving problems, even in a traditional notation, do they do so spontaneously when checking their answers or when trying to overcome an impasse? The study reported in this paper examines this question, describing the situations where students solving problems reasoned solely about the notation as well as those where students reasoned about the referents of the notational constructs.

This study was conducted in basic microeconomics, supply and demand problems more specifically. The study consisted of four groups of students solving a microeconomics problem. The first two groups solved the problem using supply and demand graphs, then solved

an associated set of equations, and finally were asked to explain why their prediction would happen in the economy. Another two groups used the software product STELLA to develop a model of the situation and to test it. Finally, these students, like the traditional groups, were asked to explain why their prediction would have happened. As it turns out, the students experienced great difficulty using STELLA, so those results will not be reported.

In the next section, we present a brief introduction to microeconomics. To describe the concepts used in this study we will show a correct solution to the experimental problem. In later sections, we describe the study itself, and its results.

Economics background

We chose economics as the domain of inquiry for this study because many people have intuitions about the ways economic factors interrelate in real world situations (Salter, 1986). Further, people have everyday experience with economic relationships, and are confronted with economic issues in much of the popular press (Huskey, Jackstadt, & Goldsmith, 1991). Microeconomics also has a well defined set of notations that all students use. Thus, microeconomics allows us the opportunity to look at the interrelationship between notational and situational reasoning.

To concretize this discussion, let us first present some basic economic theory and how the problem used in this study should be solved. This section is intended as a simple introduction to the economics concepts used in this study, rather than a complete overview of the field.

Many people are familiar with the basic concepts of supply and demand. The Law of Demand states that as price decreases, quantity demanded increases. This captures the general idea that people will buy more of a product, or buy the product more often, as the

price of the product decreases. There is a similar idea about the producers of the goods consumed. The Law of Supply states that firms will produce more of a good when its price rises.

These two laws taken together define a market made up of consumers and producers. At any given price, some number of people want to buy some amount of the good, the quantity demanded, and some number of producers are willing to make some potentially different amount, the quantity supplied. If the quantity demanded equals the quantity supplied, as in the situation shown in Figure 1, the market is at equilibrium. If more of the good is desired than is made at some price, then there is a shortage. If the situation is reversed, with more of the product being produced than consumed, there is a surplus.

Economists hypothesize that if there is a shortage, then suppliers will be able to charge more for a good. The Law of Demand predicts that this increased price would lead some of those who would have bought at the lower price not to purchase. The competing drives of the suppliers and purchasers will eventually cause the price to settle to its equilibrium level where the quantity demanded equals the quantity supplied. This equilibrating process can be thought of as sliding along the supply and demand curves. That is, in a shortage, the suppliers can inch the price up, bit by bit, until there are no people who would still purchase the good at a higher price. Equilibrium, where no extra goods are produced and no consumers are left wanting, is shown in Figure 1. Adjustment to move towards equilibrium is shown in Figure 2. However, the adjustment shown in Figure 2 does not achieve equilibrium. Instead, the market displayed remains in a shortage. More goods are produced as the price increases, and quantity demanded decreases. However, the two are not equal yet; thus, additional price increases should take place.

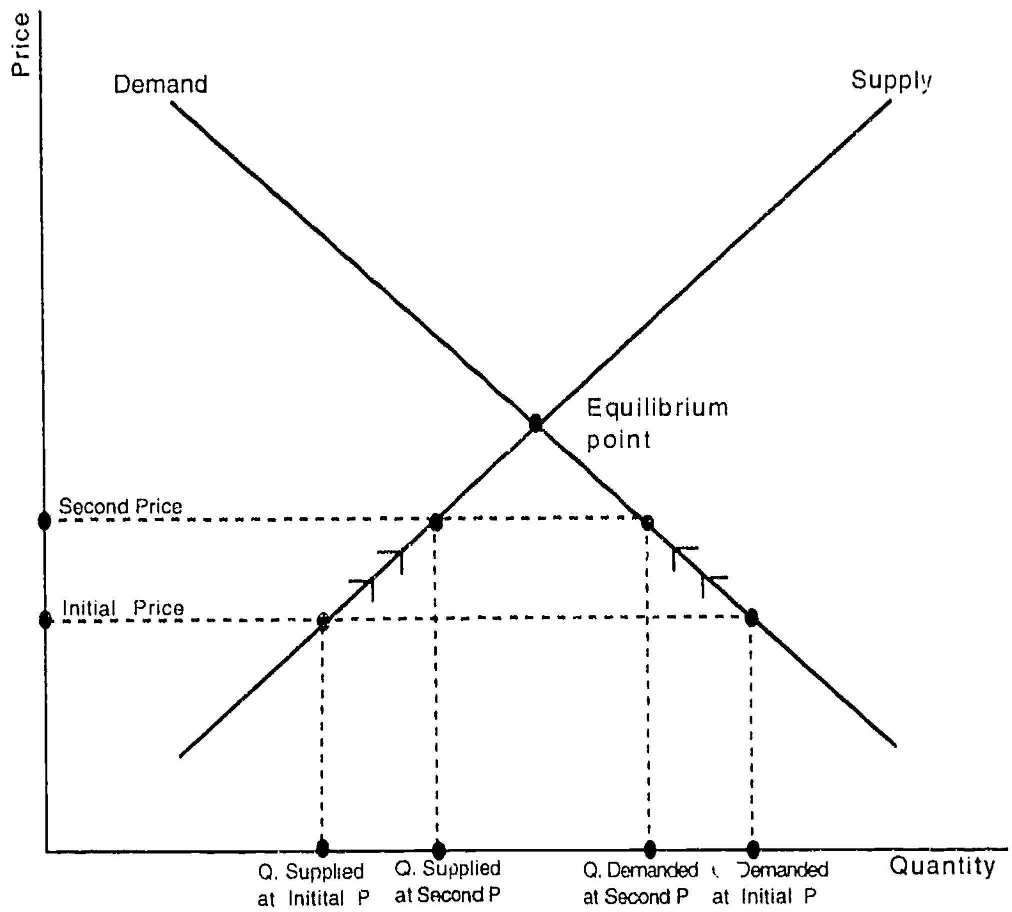

Figure 2: To achieve an equilibrium price, suppliers and consumers negotiate to adjust the price until there are no extra goods and no consumers who would buy the product at a higher price. Not all situations are at equilibrium. This graph shows a disequilibrium.

Describing these processes in the context of a real problem may help clarify this. We used a problem in this study modified from Strober and Cook (1992). This "teacher problem" described a severe teacher shortage at a small city school district, and proposed a 20% pay hike as a way of alleviating it. Following the lead of traditional microeconomics texts (e.g., Samuelson. 1980), we broke the teacher problem into graphical and equation portions. The text of the graphical component of the problem follows.

> The Butler, Pennsylvania, school district has a significant shortage of science teachers, and is attempting to remedy this situation. A school board member has proposed increasing the science teacher's pay rate by 20%. What effect will this have on the number of science teachers in the Butler schools the following year? In later years?
>
> Use a Supply and Demand Graph to predict the change in the numbers of teachers in the schools.

In Figure 1. the axes are labeled "Price" and "Quantity". This problem refers to the price of teachers, which is their salary. Thus, salary is the price component of this problem. Further, quantity is the number of science teachers in the Butler school district. The teachers themselves are supplying the desired commodity, and the school board demands some quantity. With these analogues in mind, let us now describe a correct solution to this problem.

The initial state of the problem would be constructed from the proble. description. and is depicted in Figure 3. Developing this initial state requires significant reasoning, and represents a major portion of the problem solution. The Supply curve has a positive slope, indicating that more teachers would be willing to take positions in the Butler school district

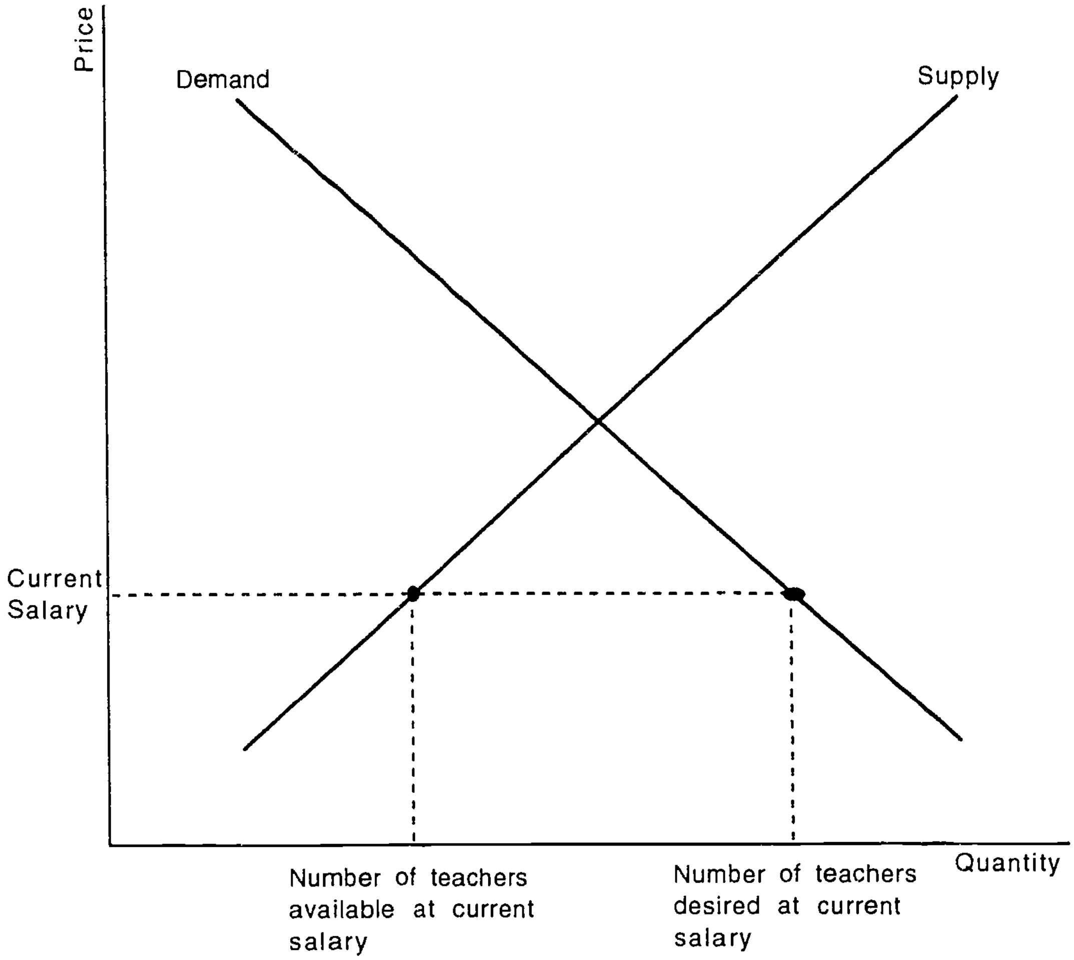

Figure 3: The starting state of the teacher problem. Notice that there is a shortage — fewer teachers available at the starting salary rate than slots to be filled.

as the pay rate increases. In contrast, the Demand for teachers on the part of the school district reduces as the salary increases, since the school can only afford to spend a certain amount total for teachers. If the salary rises too much, the district will begin to reduce the number of teachers it requires, perhaps by increasing class sizes.

This problem requires reasoning not only about shortages, but also about another key distinction in microeconomics, short run versus long run effects. Recall that the problem

asked about later years. This refers to long term effects. The important difference between short term and long term effects is the ability of firms to enter or leave a market. This difference plays out in the graphical notation as well.

It takes time to set up a new plant or modify an already existing one to produce new good. Thus, in the short run, additional producers of a good can not enter the market, no matter how profitable the market might be. However, in the long run, additional firms can adjust their plants or build new ones to compete with the suppliers in the market. Since there are now additional suppliers, there will be more goods available at each price level. Such a situation can not be described by sliding along a single supply curve, because that will not indicate more units available at each price.

As mentioned above, the Teacher problem has a short term component and a long term portion. In the short run, there will not be sufficient time for more potential teachers to enter education schools and receive teaching licenses. This suggests that any new teachers in Butler due to its salary increase will be coming from people who are already licensed to teach, rather than leading to a greater number of people in the world who are able to teach. To rephrase, there are no additional suppliers, but rather the existing firms (teachers) are producing more from the standpoint of the Butler district, by changing school districts. Thus, the only way to note this on the diagram is through a simple change in the salary rate determining the new quantities of teachers available and desired, but with no changes in either the Supply or Demand curve. This situation is shown in Figure 4. Since no numbers are yet available in the solution, there is more than one possible solution. For example, the salary change could result in an equilibrium state, where supply equals demand. Alternatively, the salary increase could be too much, leading to a surplus of

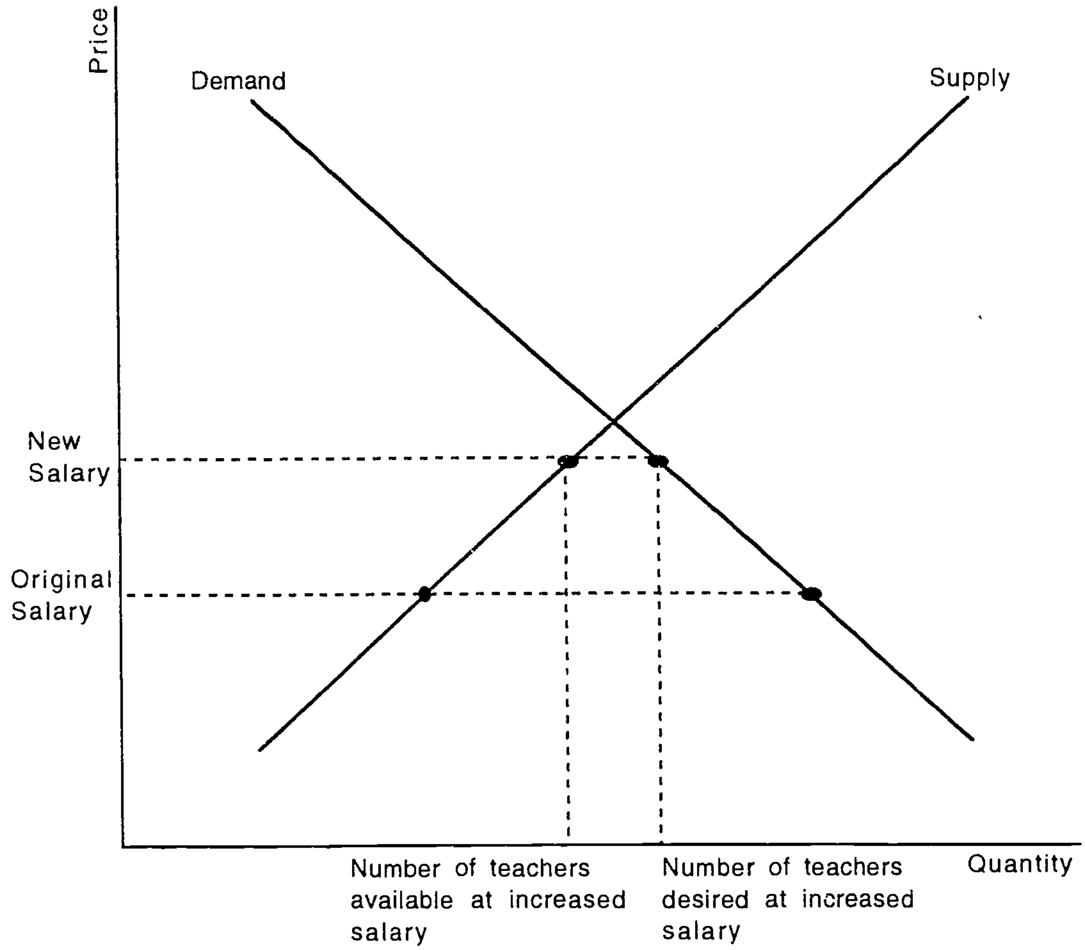

Figure 4: A representation of the market after the salary increase. Notice that there are more teachers available, and that the school district has scaled back its needs as well.

teachers. Finally, there could still be a shortage, just a smaller one, after the increase, as in Figure 4. This portion of the problem is indeterminate, as are many microeconomics problems (cf., Samuelson, 1980). The constraint to one solution does not occur until the equations describing the mathematics of the relationship are introduced. The equations are described later in this section.

Since moving along a supply curve will not depict long term effects, another notational convention must be introduced. Additional supply is depicted by moving the supply curve

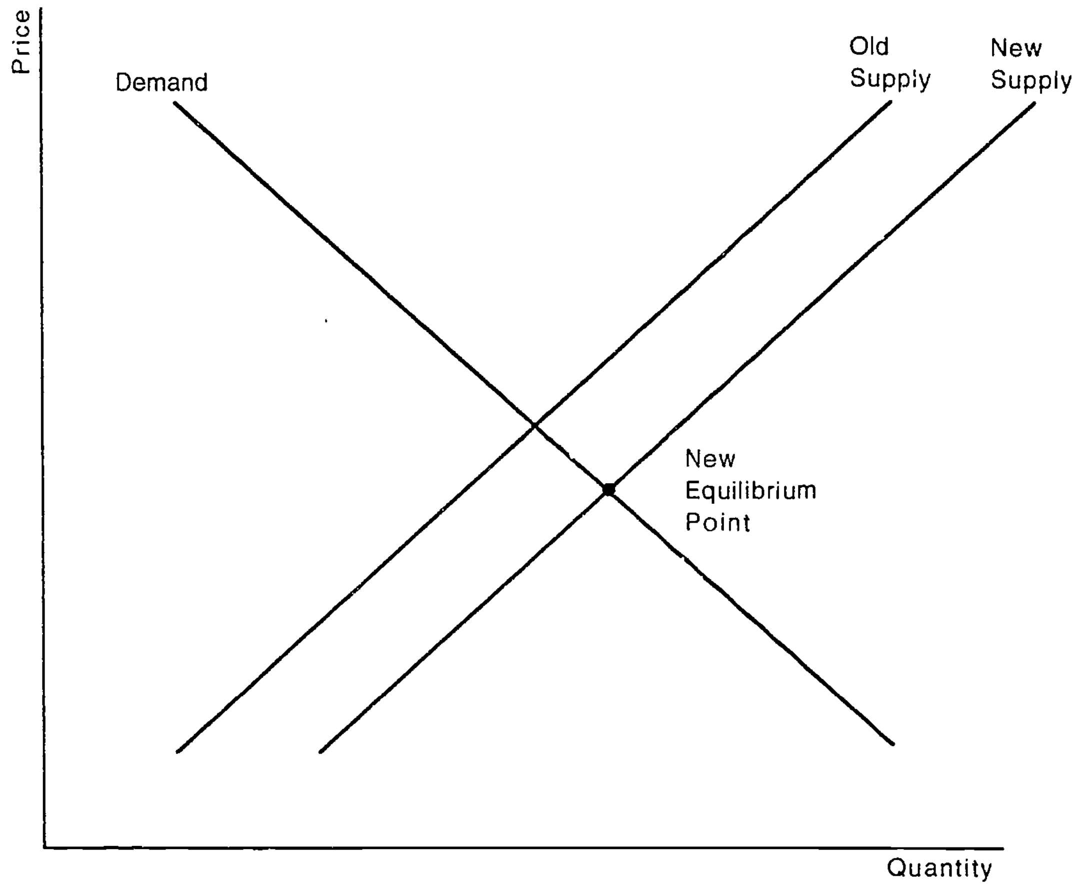

Figure 5: In the long run. firms can enter or leave the market. so the supply curve itself can shift.

to the right. This situation is called as shifting the supply curve. A shift of the supply curve to the right. as in Figure 5. indicates that more firms have entered the market. Sliding along curves. as in Figure 2. occurs in the short run. as existing producers and consumers adapt to the market prices by making or buying more or less.

The final phrase of the teacher problem asks about the effects of the salary increase in later years. This portion of the problem concerns long run outcomes. In the long run. rather than simply move along static curves. as shown in Figure 2. the market itself can change. For example. in the long run. people who are not currently teachers could enter

the market, since they would have had time to attain teaching certificates. In other words, more suppliers could enter the market for teachers. Thus, there could be a shift of the supply curve, as shown in Figure 5, in the long run.

Figure 5 does not match the situation in the teacher problem completely, however. School districts do not need an infinite number of teachers; at some point the additional costs of adding teachers is greater than the benefits of hiring them. For example, it probably is not cost effective to have more teachers in a school district than there are students. The Butler school district will only need a certain number of teachers. Thus, the district's demand curve will not shift, but rather exhibit a cessation of demand when some quantity of teachers have been hired. This situation is shown in Figure 6.

Supply and demand graphs display a market in equilibrium or disequilibrium, and can show changes in number of suppliers in long run effects. However, it does not show what caused the prices to increase, nor does it display why additional firms entered the market. The notation does not capture the reasoning that was used in its creation. Thus, if students need to reason about the situation, as they did during the creation, from the notation, they must recreate it entirely. Such a situation might arise when students hit an impasse in the problem solving. Students might also be able to use their understanding of the notation itself to try an overcome impasses without reference to the situation embodied by the graph. In this study, we will examine the situations in which students overcome impasses via situational reasoning versus those where the students use notational manipulations to overcome the impasse.

As mentioned earlier, traditional microeconomics problems often have two phases, the graphical phase much like what was just described, and an algebraic phase (cf., Samuelson,

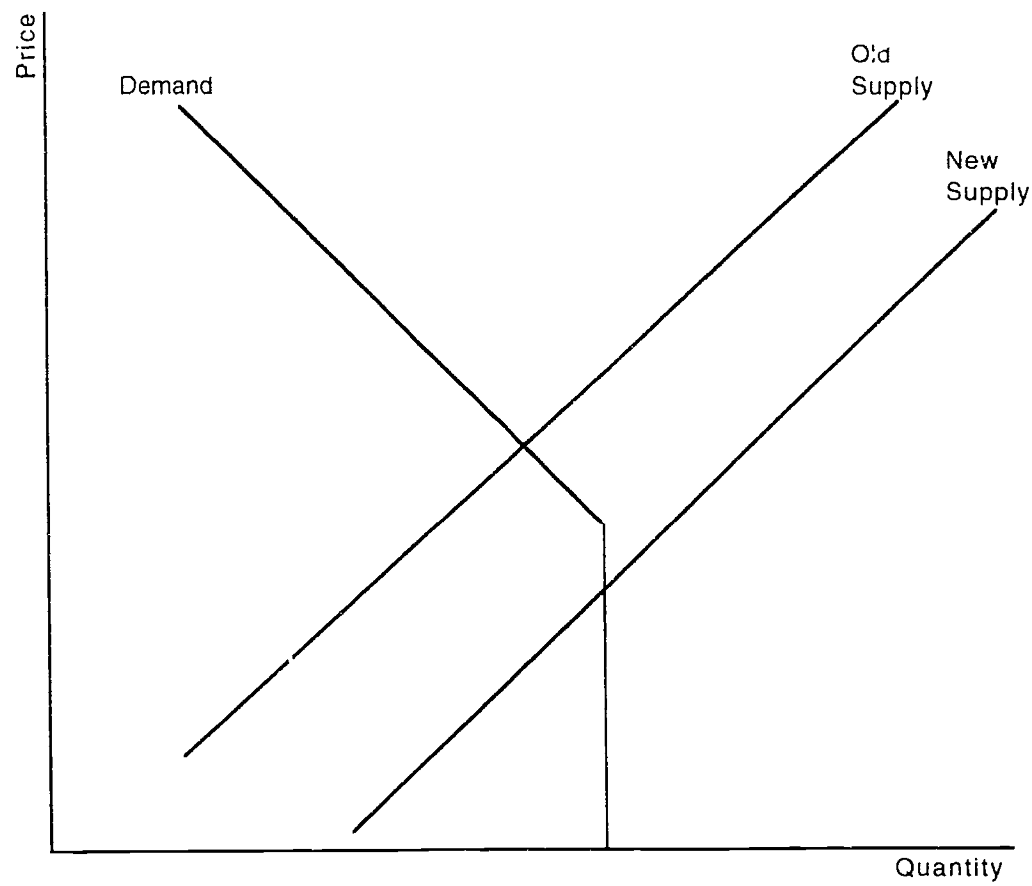

Figure 6: A representation of the long run effects of the salary increase. Notice that after a certain number of teachers are hired, the district will not hire more, no matter how inexpensive the salary becomes. This cessation of demand is represented by the bend in the Demand curve, which shows that after a certain quantity is achieved, no more teachers are required.

Initial Salary = \$27.00

$$P = 120 - 3Q_D$$
$$P = 15 + 4Q_S$$

$$\begin{aligned} 27 &= 120 - 3Q_D \\ -93 &= -3Q_D \\ 31 &= Q_D \\ 27 &= 15 + 4Q_S \\ 12 &= 4Q_S \\ 3 &= Q_S \end{aligned}$$

Post-raise Salary: \$32.40

$$P = 120 - 3Q_D$$
$$P = 15 + 4Q_S$$

$$\begin{aligned} 32.40 &= 120 - 3Q_D \\ -87.6 &= -3Q_D \\ 29.2 &= Q_D \\ 32.40 &= 15 + 4Q_S \\ 17.4 &= 4Q_S \\ 4.35 &= Q_S \end{aligned}$$

Table 1: A solution to the equation component of the teacher problem. The top set of equations refers to the original salary, and the bottom set refers to the increased salary. Notice that there is a significant shortage at both the original salary and the increased salary.

1980). In the algebraic phase, students are typically given equations to model the situation they just described graphically and are asked to solve for the equilibrium quantity or the equilibrium price. We asked students to determine, using a set of equations and initial values we provided, the initial quantities in the market, and those quantities after the 20% price increase. The algebraic portion of the problem follows. The solution to the problem is shown in Table 1.

Suppose the demand curve for teachers is

$$P = 120 - 3Q_D$$

where P is the salary per year and Q_D is the quantity demanded per year of teachers.

Further, suppose that the supp'. curve for teachers is

$$P = 15 + 4Q_S$$

where P is still the salary per year and Q_S is the quantity of teachers supplied.

If teachers' salary rate before the increase is $27 per hour, how many teachers will be supplied and demanded? Then, what will happen after the 20% raise in salary rate (to $32.40)?

Again, as in the supply and demand portion, there are mappings between the equations and the situation they describe. The problem description indicates that there is a shortage at the start of the problem, and this is confirmed by the algebraic solution, shown in Table 1. Furthermore, the equations indicate that the short run outcome of the salary increase is not very impressive, since it decreases the shortage only slightly. Students need not tie the algebra into the situation at all, however, since they are by and large quite proficient with algebraic manipulations. Thus, students could solve the equations, yet never update their understanding of the situation based upon the new knowledge.

This section has described two common components of microeconomics problems, the supply and demand graph and associated equations, and shown a correct solution to both parts of the teacher problem.

The teacher problem was given to four groups of middle school students. Two of the groups used STELLA, a computer program used for developing causal models of behavior. The other two groups used traditional supply and demand graphs as described here. Because of equipment difficulties and difficulty learning to use STELLA, we shall report data only from one supply and demand group.

The discussion of microeconomics provided here is intended to make the discussion of students' actions clearer, and to provide a correct reference for use when we discuss the students' difficulties. This study investigated the role of situational knowledge in identifying and overcoming impasses during problem solving in applied contexts with a traditional notational system. In this study, we shall focus our attention upon locations where students experienced difficulty solving the problem. In these locations, we examine students' strategies for overcoming the difficulty, and whether it was overcome successfully. We will argue that the students could use strategies for operating on the notation without reference to the situation to overcome the impasses or could fall back on reasoning about the situation itself, and that these two strategies resulted in different learning outcomes.

Method

Subjects

Fourteen students (nine males and five females) participating in a summer camp for gifted seventh through ninth graders took part in this study. All students volunteered to take part and received no payment. The students completed several weeks of an introductory economics course at the camp, and so had significant familiarity with economic concepts like supply, demand, shortages, and surpluses.

All students completed the task. However, due to equipment difficulties, one of the two traditional groups was excluded from analysis. Further, students experienced so much difficulty using STELLA that most of their sessions were spent not on economic reasoning, but rather on trying to understand STELLA itself. Thus, we have chosen to focus our analyses on the remaining traditional group.

Materials

The students solved two problems during the sessions in groups of three or four. These problems used microeconomics issues to address policy concerns, and were constructed from economics textbooks. The problems concerned supply and demand relations, and required reasoning about surpluses and shortages. The problems are displayed in Appendix A. The Traditional group used a whiteboard to draw supply and demand graphs, and to solve equations given in the problems.

To form each problem, we added one requirement to the traditional two part economics problem. First, students read a description of a situation and a proposed solution, and made a prediction of the effects of the solution, using a supply and demand graph to create this prediction. Then, after coming to an answer as a group, the students solved a set of equations that modeled the problem they had just solved graphically. Finally, after completing the equations, the experimenter removed the solution, by erasing the whiteboard, and then asked the students to "...explain why you got those numbers." This probe was intended to see how the students would describe the process leading to the outcomes they achieved.

Procedure

The students first chose a partner. Then the experimenter joined these groups of two with unmatched people to form groups of three or four for the procedure. The students solved all problems in groups. We did not assign students to groups to assure gender mixed groups, and so ended up with one predominately female group, with the remaining three groups all male. The group discussed in this paper was made up of three males.

These groups were randomly assigned to either the Traditional or the STELLA condition. Each group went with one experimenter to different areas. Each group worked in an enclosed area by themselves, and was not aware of the activities of the other groups.

After being led to their experimental areas, the groups were told that they would be solving some economics problems much like the ones they solve in class, only they were to do it cooperatively in groups. The experimenter then solved the first problem (Problem 1) with the students to show them the experimental procedure, and then gave them one to do alone (Problem 2). All these interactions were videotaped.

After completing the task, all students were debriefed about both conditions in the study and the Traditional students were given the opportunity to play with STELLA if they wished.

Results and Discussion

In this section, we will describe the ways students' real world knowledge interacted with their formal problem solving. More specifically, we shall examine the instances where students ran into problems and see how they overcame the difficulties.

Recovering from impasses is a major component of problem solving, and a primary

source of learning (Chi, Bassok, Lewis, Reimann, & Glaser, 1989; Lewis & Anderson, 1985; Schank, 1986). Merrill and Reiser (1993) argued that traditional notations can interfere with recovering from impasses because the notations may not support students' reconstruction of what they were trying to do and could even constrain the students' actions. These constraints might suggest actions to the student that will repair the symptom of the impasse, but do not match the more important situational constraints. Since we are interested in the ways students use notational and situational reasoning in problem solving, a key place to look for interference between the two is during impasses. Thus, we will focus upon the ways students recovered from impasses that occurred during problem solving.

Although we use the term impasse, we are not using the term in its common parlance, but rather in a logical extension of its meaning. Impasses are problem solving situations where students experience difficulty achieving a correct action (VanLehn, 1988, 1990). A resolution to an impasse gets the student past the difficulty. Note that the resolution does not have to be correct, but rather must allow the students' problem solving to continue. Thus, an impasse resolution could be incorrect.

We defined an impasse as a problem solving situation where the group required several verbal interchanges to decide how to proceed in the problem solving. These disagreements or confusions within the group about the next action were identified from the videotaped interactions. After finding each impasse, we categorized its resolution into one of two possible types: notational reasoning or situational reasoning. A resolution was classified as using situational reasoning if any of the utterances involved in attaining a resolution referred explicitly to an element of the situation. For example, a student utterance that referred to teachers, schools, or salary would be classified as situational reasoning. However,

Table 2: The five impasses our students encountered while solving the Teacher problem

1. How to represent the salary increase
2. Does the supply curve shift as well
3. What about in later years
4. Why is there a shortage at the start
5. Using the language of the situation

if students used the terms that are associated with the notation. such as cost. suppliers, or consumers. we categorized that as notational reasoning. It is. of course. possible that students were reasoning about the situation even though they used the notational terms. However. we chose not to classify an utterance as situational reasoning unless there was clear evidence that such a classification was correct.

We will next describe the impasses the students encountered and how they were overcome. As will be seen. sometimes problem solving was able to continue correctly past an impasse. while in other circumstances the students' repairs designed to overcome the impasses resulted in errors. We argue that when the students used reasoning about the situation embodied by the problem. they developed a successful repair. but when they relied upon the notation. they constructed a patch that was incorrect.

The diagrammatic portion of the teacher problem

Impasse 1: How to represent the salary increase

The students in the analyzed group began solving the teacher problem by drawing the supply and demand curves. as shown in Figure 1. However. instead of starting with a shortage, as in Figure 3. they attempted to begin the solution in equilibrium. In fact. they

did not even think about the initial setup of the graph, but rather just drew the supply and demand graph as they had in every other problem.

> Student 1: OK, let's put our philosophy up there [pointing to whiteboard]
> Student 2: Yup, here we go.

The unfortunate aspect of this rote creation of the diagram is that the group did not stop to recall that the initial situation was a shortage. Novice economics students have a great deal of difficulty reasoning about shortages, particularly in terms of problems that do not begin in equilibrium (Strober & Cook, 1992), as does the teacher problem.

This difficulty may arise from the fact that many supply and demand graphs begin at equilibrium, and require students to perform some action to make the equilibrium change. Students will often learn to use rote procedures such as keywords in word problems to select solution methods rather than reasoning about the situation (Fuson & Willis, 1988). The students followed their well learned procedure for solving supply and demand graphs — start the graph in equilibrium and then change something to create the disequilibrium. In this case, they had to shift either the supply or demand curve to create the disequilibrium.

The students decided to shift the demand curve, asserting in effect that the demand for teachers has suddenly changed.

> Student 2: OK, OK, I think the demand will go up to here [draws new line on supply and demand graph] Student 1: Yeah, OK, the demand goes up. Right.

After drawing the new line, shown in Figure 7, the diagram indicated that teacher salaries would have to increase to achieve equilibrium at the new demand levels. Thus, their diagram showed that an increase in demand had caused the pay increase. However, the problem had stated that the school board decided, without any market mechanism involved, to increase the pay rate 20%. That is, the pay raise in the problem was not

caused by an increase in demand. but rather by a shortage that already existed. The students. recognizing that the diagram already indicated a pay raise, had encountered an impasse: How should they account for the pay raise?

> Student 1: Hey. wait ... what about the salary increase? Do we increase it again?
> Student 2: No. it already went up 20%. That's what the problem says. right?

The students decided to rely upon the information in the diagram. which indicated that the pay increase had come about due to a sudden increase in demand. rather than to refer back to the situation. which indicated the pay raise was a short run decision of a school board. The next utterance shows a second student arguing that the diagram was fine by saying the pay raise was a market force.

> Student 2. Oh. the pay raise. automatically ... it automatically goes up. It's not a separate thing.

A resolution to an impasse just has to let problem solving continue. The impasse here was that the students found the pay raise happening automatically. when they expected to perform some graphical operation to cause it to occur. The resolution was to declare that expectation wrong, by asserting that the pay increase was caused by the sudden increase in demand. This resolution relies upon the information represented in their diagram — the shift in the demand curve — rather than upon reasoning about the situation. Recall that the demand curve can only shift in the long run. indicating a sudden increase in the market size. In this case. a demand curve shift would require building new schools. which typically takes several years. Had the students fallen back on situational reasoning in this case. they would have been much less likely to shift the demand curve since the situation was in the short run. and so they might have reconsidered their conception that the problem began in equilibrium.

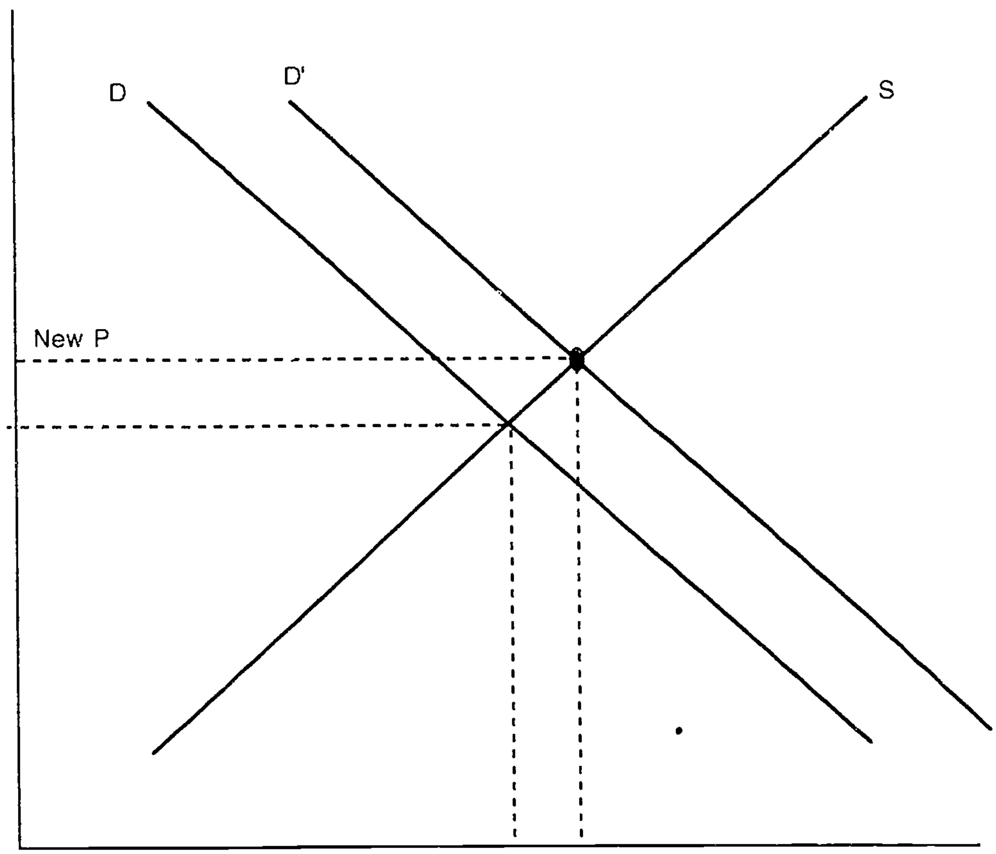

Figure 7: The students' solution to the graphical component of the teacher problem.

Impasse 2: Does the supply curve shift as well

Contrast this strategy for overcoming this impasse with that for the next one encountered by the students. After having moved the demand curve, the students wondered if the supply curve would also have to shift. Again, in many supply and demand problems both curves are moved in response to some long run change in the market.

Student 1: OK, what about the supply curve? Does it have to shift, too?

Recall that the supply curve can not shift in the short run, since curves shifting implies a change in the underlying structure of the market. In this case, a shift in the supply curve would indicate that there were more teachers in the world. Clearly, in the short run, this is not true, because aspiring teachers must get a license to teach, which often requires several years of school. Thus, in the short run, teachers can neither be created nor destroyed, but can only change districts.

Thus, reasoning about the situation in this impasse would indicate that the supply curve should not be moved. This impasse appeared to give the students much more difficulty than the previous one. They struggled with it for a few minutes, with many long pauses, umm's, uh's, and other utterances that indicate confusion.

Student 3: Wait, no, don't move the ...uhh ...Should it [the supply curve] shift too?
Student 1: Umm, I ...uhh
Student 2: No, there aren't more teachers, there's the same number.
Student 1: Right, it shouldn't shift.
Student 3: But there are more teachers, right? [points to graph]
Student 1: In the *district*, in the *district*, not more overall!
Student 3: Oh, yeah, you're right.

Finally, the students did fall back on reasoning about the world as they understand it, asserting that the number of teachers in the world is not changed by one school district

increasing their salary, but the number of teachers working for the Butler school district could increase, due to the hiring of "[Student 1] ...teachers from around there." This rationale enabled the students to decide upon an action that would end the impasse. In this case, the action was not to move the supply curve.

Thus, students facing a more serious impasse did fall back upon situational reasoning, and were successful in resolving the impasse. In this case, the resolution was not to perform an action that many problems would have required — shifting the supply curve. They decided not to shift the curve by thinking about the fact that one district's salary increase will not lead to a general increase in the pool of teachers; in other words, the students achieved a correct resolution by reasoning about the situation embodied by the problem. In contrast, the earlier, somewhat easier impasse elicited notational reasoning and did not result in as successful a resolution.

Impasse 3: What about in later years

This generalization that notational reasoning does not lead to as successful resolutions holds true for the final impasse in the graphical phase. However, this final impasse represents a counter point to the previous ones. In this impasse, students were able to achieve a correct prediction of the events, but were unable to express it correctly.

The students, having dealt to their satisfaction with the effects of the pay increase in the next year, turned to the portion of the problem concerning effects in later years, which are long run effects. A supply and demand graph does not provide much support for reasoning about longer run outcomes. The only information left explicit in such a diagram is that one curve or the other has moved, but nothing about why it shifted. While this is an acceptable product of problem solving, the process by which the solution is achieved is the goal of the

problem solving (Heller & Reif. 1984). To determine the long run outcomes of a change, students must determine if more people will try to get teaching certificates. and so forth. This sort of reasoning should be performed during the creation of the solution. However, a solution to a long run question can be answered by choosing a curve to shift with minimal explanation of why the situation calls for that curve to be shifted.

That was not the case here. The students in this study concentrated on the long run decisions of potential teachers and future school boards. For example. students thought about factors not explicitly mentioned in the problem.

> Student 2. Nothing will happen [in later years].
> Student 3. Until inflation rises enough so that the pay raise becomes too little.
> Student 2. Well, yeah, but maybe they [the school board] will keep giving them [the teachers] cost of living increases.

This interchange represents hypothetical reasoning about the effects of a factor not mentioned in the problem at all, inflation. Inflation is quite relevant, however, since an increase in the cost of living would lessen the impact of a one time salary increase. Such hypothetical reasoning is often regarded as a major goal of social studies education (Barth, 1984; Stigler. 1983; Voss. Blais, Means. Greene, & Alwesh, 1986), so its appearance is an indicator of successful problem solving.

The students also recognized that there were only a limited number of teachers required for the Butler school district.

> Student 3: But they only have a limited number of spaces.
> Student 1: Yeah. when they get filled up. they won't want any more.
> Student 2: So demand will go down ...
> Student 1: Way down.
> Student 3: It'll go away entirely.
> Student 1: Right, so it'll go way down to here!

This led them to try to represent the school district's future lack of desire for additional

teachers through a drastic shift in the demand curve, as shown in Figure 8. This diagram is not the correct means for noting such an inference. The correct means for recording this inference is shown in Figure 6. Thus, the students took a correct prediction, that the schools would not want more than some certain number of teachers, and made an error when trying to translate it into the official notation. Thus, even though the students were able to come up with correct predictions incorporating real world knowledge not directly stated in the problem, that school districts only need a certain number of teachers, they were unable to record this inference in the traditional notation.

Summary of the graphical phase

During the graphical phase of this problem, the students encountered three impasses that varied in severity. The students achieved correct solutions for some of the impasses, and were able to deal with others to their satisfaction, although incorrectly.

The impasses these students could easily overcome were resolved by using notational reasoning -- removing the impasse by manipulating the diagram in some manner without referent to the situation of the problem at all. However, this notational reasoning did not necessarily lead to success, as in the demand shift example, which resolved the impasse for the students but did not do so in a manner consistent with the situation.

The students resolved more difficult impasses by falling back upon everyday experiences. The students knew a great deal about the world, such as that districts have limited positions for teachers and that inflation affects the importance of pay raises. This situational reasoning allows students to access this knowledge, which can help constrain the search for a resolution to the impasse. Indeed, when these students attempted to reason about the situation, they achieved more success and generated rationales for outcomes that they had

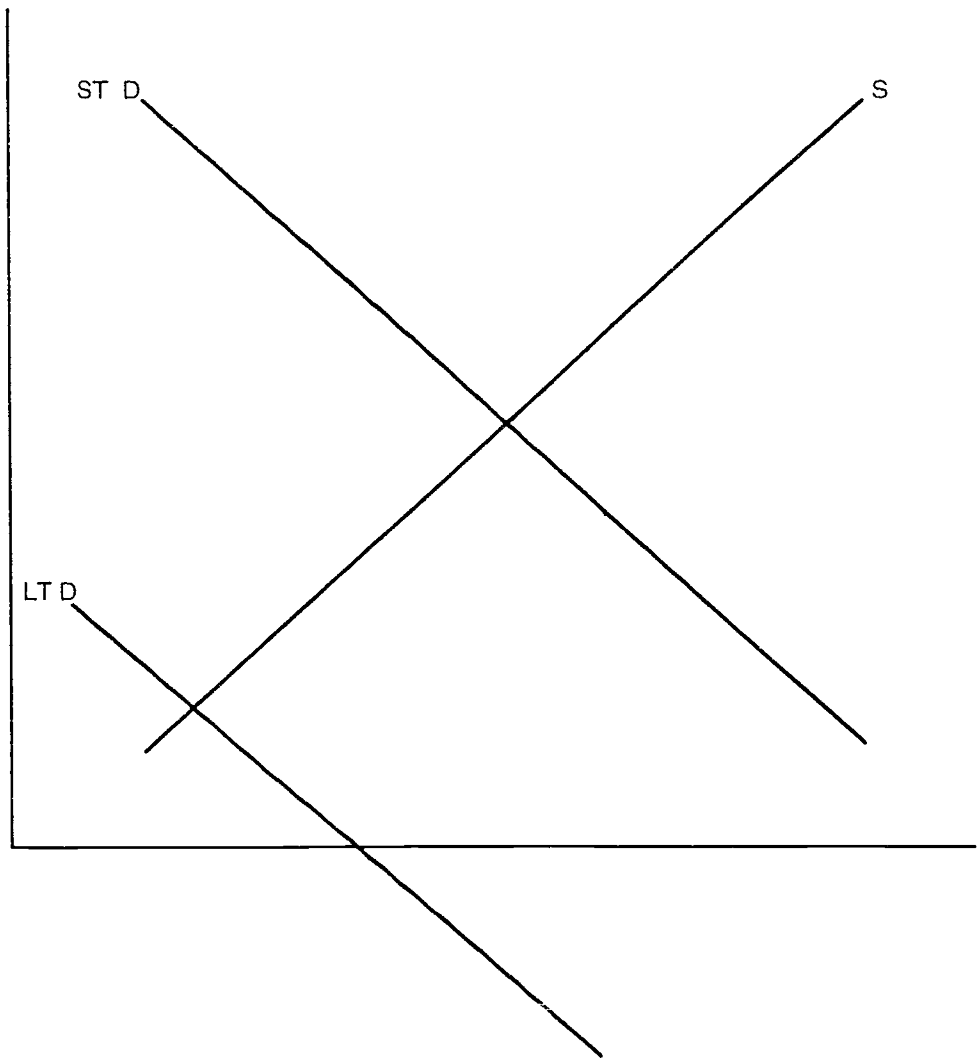

Figure 8: The students' notation of a long run demand decrease.

not previously considered.

Equation component of the teacher problem

Impasse 4: Why is there a shortage at the start

Despite the generalization so far that reliance upon notations leads to less successful outcomes, we must point out that notations are useful when used in conjunction with situational reasoning. The next impasse indicates the advantages of relying upon notational reasoning rather than only upon beliefs about the situation.

These students were quite able to solve the algebraic manipulations, and their answers were mathematically correct. The only impasse during the mathematical phase concerned students' attempts to comprehend the numbers resulting from their equation solving. The correct solution to this portion of the problem is shown in Table 1.

The mathematics indicated that the situation begins with a significant shortage. However, recall that the students' initial solution as given in the diagram phase, did begin in equilibrium.

Recall that we argued that the students began their solutions in equilibrium because many economics problems begin there. Similarly, the equation portion of those problems starts out in an equilibrium status. The fact the mathematics indicated there should have been a shortage initially confused the students considerably, leading to a significant impasse. The students immediately concluded that their algebra must have been incorrect.

> Student 2: Whoops! Wait, something's wrong here.
> Student 3: Why?
> Student 2: 'Cause they only have this many [points to quantity supplied] but they want this many [points to quantity demanded].
> Student 3: Oh, there's too few [teachers].

After checking the mathematics. and deciding that it was correct, the students then asserted "[Student 1.] OK. this is a different problem. remember." Rather than rethink their initial conception that this problem. like many others. began in equilibrium. these students chose to declare the second part of the problem completely disconnected from the first. although they were specifically instructed that it was the same situation.

This resolution did not appear to be satisfactory to the students. however. One of the students asked whether the problems were supposed to be the same, and was told that the equations modeled the same situation as the diagram. This left the students facing a problem: Their conception of the situation must be resolved with the apparently correct mathematics. Despite their initial willingness to separate the equations from the graphical component, they quickly reconciled the mathematical notation with their conception of the situation by recalling the problem. "[Student 3.] it [the problem] said there was a shortage." Thus. there should be a shortage at the original salary rate, because the problem starts with a shortage.

The mathematics solution indicated an initial shortage, even though students had developed a conception of the model that began the situation in equilibrium. Initially, the students separated the two into different problems. so that the conceptions did not clash with each other. However. when the experimenter prompted them to reconcile their algebra with their conception about the situatio: they were able to form a correct understanding of the situation that reflected the initial shortage.

Once again reasoning about the situation led these students to more successful impasse repairs than focusing upon the notation or relying upon it to explain an outcome. However, these impasses highlight the fact that notations are also crucial, since the notation may be

able to reveal portions of the situation that students have not heeded sufficiently. Thus, we can not focus solely upon notations nor upon situational reasoning, but rather we must lead students to attend to both together, using the one to constrain the formation of the other.

There was no notation used in the final component of the task, but this distinction between representation and situation continues nonetheless. The final portion of the task required students to describe their understanding of the situation and why they made the predictions they did.

Explanation portion of the teacher problem

Impasse 5: Using the language of the situation

The students were quite willing to explain why they predicted the outcomes that there would be a shortage after the salary increase. There was one interesting impasse in their description, however.

When initially asked to explain their answers, one of the students answered "[Student 3] 'Cause it costs more over here [points to pay raise section]. They can charge more, so they want less, but can get more." This answer is phrased in terms of traditional microeconomics terms, costs, charges, and so forth, but does not refer to teachers or use language one typically uses to refer to people. This appeared bothered the other members of the group, who interrupted to say "[Student 2] Wait, we're talking about teachers here!"

After this interruption, the group together developed a story involving the actors in the situation, the teachers and the board, and the decision processes of each.

> Student 2: The school has to pay more money for each teacher, which makes them want to get fewer, maybe by increasing class size or eliminating some

classes. But since the teachers can get more money for teaching, they are more willing to take a job there, since they can get more ...
Student 3: Yeah, the board can't spend too much, but the teachers are happy.

During this more contextualized description, students thought of factors possibly relevant in the long run that they had not considered earlier, and explicitly referred to the additional steps that might need to be taken by the board to reduce their personnel needs.

Student 1. Even after the pay raise, there's still a big shortage. I guess the change isn't big enough.
Student 2. Well, teachers don't get paid much anyway.
Student 1. Yeah, they [school board] will have to raise their [teacher's] wages a lot to get enough.

We argue that this hypothetical reasoning arose because students were trying to tie their understanding of the world to their problem solving predictions. The students knew that people in general would rather have more money than less, but that there are limited budgets for paying them. Further, they were able to make inferences about teacher's preferences from the mathematical portion of the problem to recognize that a much larger salary increase will be required to wipe out the shortage. Again, this suggests that students may need to employ both their notational knowledge and their situational knowledge to be most successful at solving problems, and that each source of knowledge may serve different purposes.

Conclusion

This pilot study was designed to generate hypotheses about the role of external representations in problem solving, particularly focusing on how students are able to overcome impasses. We argued that when students attempted to overcome impasses using simple manipulations of the notational scheme, they often lead themselves into more difficulty.

Contrast this with those cases where students fell back on reasoning about the real world situation represented by the problem, in which they experienced more success with the repair.

A patch based upon reasoning about the situation is likely to be more robust than one based solely upon notational manipulations (cf., Ohlsson & Rees, 1991), because of the representational power of real world knowledge. People know a great deal about everyday life, far more than they know about algebra in general. Using this knowledge allows the students to make connections to relevant information that is not explicitly represented in the problem. However, notations are clearly not completely useless, since a notation can help students identify errors in their understanding of a situation.

It is clear, therefore, that instruction must not focus solely upon notational procedures, but also must not ignore notations in favor of solely attending to students' understanding of the situation embodied by the problem. Both must be used in conjunction to ensure students select and employ appropriate solution processes. Experts employ both informal reasoning strategies when deciding how to solve difficult problems, and use notational strategies to check their progress (Bauer & Reiser, 1990). Learning environments must support students' using of both strategies when solving problems. Unfortunately, much of instruction focuses upon the final product (Heller & Reif, 1984), which can lead students to acquire simple rote procedures for operating on notations, such as our students' procedure for setting up supply and demand problems in equilibrium. Thus, a goal of learning environment designers must be to encourage both informal reasoning about the situation in the world but also to support problem solving strategies that allow students to check their progress.

Appendix A

Problem 1 *The Clinton administration has begun a major initiative to curtail the budget deficit in the United States. One of the possible items in President Clinton's economic stimulus package was a 15% tax on gasoline and related products such as jet fuel. If the Clinton economic stimulus package does contain a 15% tax of gasoline and related fuels, will there be any impact of this tax on pollution in cities?*

Equations for Problem 1

Scientists have found that as the price per gallon of gasoline increases, slightly more people will ride buses and subways. Thus, the demand curve for public transportation is upward sloping, and the supply curve is constant. Assume the demand curve is

$$P_{gasoline} = .50 + 0.15Q_{D-public}$$

and that the supply of public transportation is constant at $Q_{S-public} = 5$ regardless of price.

If the price of gasoline is $1.00 per gallon, what will be the Q_D for public transportation? Will there be a surplus or a shortage or neither? What about at the new price of $1.15 (after the 15% tax increase)?

Problem 2 The Butler. Pennsylvania. school district has a significant shortage of science teachers. and is attempting to remedy this situation. A school board member has proposed increasing the science teacher's pay rate by 20%. What effect will this have on the number of science teachers in the Butler schools the following year? In later years?

Use a Supply and Demand Graph to predict the change in the numbers of teachers in the schools.

Equations for Problem 2

Suppose the demand curve for teachers is

$$P = 120 - 3Q_D$$

where P is the salary per year and Q_D is the quantity demanded per year of teachers.

Further. suppose that the supply curve for teachers is

$$P = 15 + 4Q_S$$

where P is still the salary per year and Q_S is the quantity of teachers supplied.

If teachers' salary rate before the increase is \$27 per hour, how many teachers will be supplied and demanded? Then. what will happen after the 20% raise in salary rate (to \$32.40)?

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